

# Open string derivation of winding states in thermal noncommutative field theories

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## ABSTRACT

The ‘winding state’ behavior appears in the two-loop nonplanar contribution to the partition function in thermal noncommutative field theories. We derive this feature directly from the purely open string theory analysis in the presence of the constant background  $B$ -field; we compute the two-loop partition function for worldsheets with a handle and a boundary when the time direction of the Euclideanized target space is compactified. In contrast to the closed-string-inspired approach, it is not necessary to add infinite number of extra degrees of freedom. Furthermore, we find a piece of supporting evidence toward the conjecture that, in the UV limit, the noncommutativity parameter plays the role of the effective string scale in noncommutative field theories.

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Open string theories in the presence of the constant background (spatial) NS-NS two-form gauge field ( $B$ ) [1], describing the dynamics of noncommutative  $Dp$ -branes, reduce to  $(p + 1)$ -dimensional noncommutative field theories upon taking a decoupling limit, where the string length scale  $\alpha' \rightarrow 0$  [2]. The resulting noncommutative field theories are known to possess remarkably stringy features such as UV/IR mixing [3] and Morita equivalence (related to string dualities) [4]. Generally, when there are compact directions in the target space, the behavior of commutative point particle field theories are markedly different from that of string theories. Considering the similar situation in noncommutative field theories, however, might reveal more of stringy features. One such example is the ‘winding state’ behavior observed in thermal noncommutative field theories [5]. Formally, thermal field theories are obtained by Euclideanizing the time direction and compactifying it with the period given by the inverse temperature. To understand this feature from the string theory point of view, one naturally has to consider a compact direction in the target space, which is parallel to the  $Dp$ -branes.

In general, there are three kinds of target space compactifications that can be of interests. The first kind is the compactification of the directions perpendicular to noncommutative  $D$ -branes. Especially in the context of noncommutative warped compactifications [6], this can be of physical importance. The second kind is the compactification of the directions parallel to the  $D$ -branes and to the nonzero  $B$  fields, which has also been actively pursued [7]. Our main interest in this note is the third kind, where one compactifies a target space direction parallel to the  $D$ -branes and perpendicular to the nonzero  $B$  field directions; we analyze the nonplanar two-loop partition function of string theory with the constant and spatial  $B$  field when such kind of compactification is present. Being the nonplanar vacuum diagram, the worldsheets in consideration have a handle ( $g = 1$ ) and a boundary ( $b = 1$ ). We show that the ‘winding state’ behavior observed in thermal noncommutative field theories [5] can be reproduced from the string theory computations on  $g = 1, b = 1$  worldsheets upon taking the decoupling limit  $\alpha' \rightarrow 0$ . An interesting point is that even the decoupled theories, i.e., noncommutative field theories, turn out to be very stringy; our perturbative string theory calculations are consistent with the proposal [8] that, in noncommutative field theories, the noncommutativity parameter  $\theta$ , that appears in the  $*$ -product between operators

$$\phi_1(x) * \phi_2(x) = \exp \left( \frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial z^\nu} \right) \phi_1(y) \phi_2(z)|_{y=z=x} \quad , \quad (1)$$

plays the role of the effective string scale  $\alpha'_{eff}$ .

The way that we derive the ‘winding states’ in the  $\phi^4$  noncommutative scalar field theory is

based on *purely* open string analysis combined with the stretched string interpretation of [11]. It is instructive to compare our approach to that of [9] based on closed string type approach. Due to the dipole effect advocated in [10], which lies at the heart of the stretched string interpretation of [11], an open string in the presence of the constant  $B$  field tends to stretch as it gains momentum, indicating the nonlocal nature of noncommutative field theories. The UV/IR mixing [3], the appearance of the IR divergence from the UV regime of the loop momentum integration in nonplanar amplitudes, can be explained by such stretched strings. Typically, the stretching length of an open string acts as the UV regulator of the nonplanar amplitudes. Since the stretching length is proportional to the momentum, the amplitudes tend to diverge as the UV length cutoff goes to zero size in the zero momentum limit. As we will see shortly, this UV/IR mixing effect plays a crucial role in the physics of ‘winding states’. As inspired by Seiberg, Raamsdonk and Minwalla [3], however, one might attempt to add extra ‘closed string’-like degrees of freedom, which, upon integrating them out, might explain such nonlocal behaviors as the UV/IR mixing and the ‘winding states’. Such attempts reported in the literature have one feature in common; we have to add infinite number of extra degrees of freedom. In the context of ‘winding states’, one has to consider an infinite number of modes that resemble the closed string winding modes [9]. In the context of the UV/IR mixing phenomenon, one can argue that the full massive closed string corrections cannot be neglected in the closed string channel description [12]. In contrast, in the purely open string approaches for the same problems, as we explicitly verify for the ‘winding state’ issue here, one does not have to add infinite extra degrees of freedom; in fact, one finds that the conventional quantized momenta (loop momenta in the particular case in consideration) of the open string zero mode parts in the compact direction conspire to produce the effective ‘winding states’. This is in line with the conventional wisdom that the dynamics that is easily described by the low energy degrees of freedom in one theory (open strings) has rather complicated dual theory descriptions (in terms of closed strings)[13].

Our analysis starts from the computation of the partition functions on  $(gb)$  worldsheets with  $g$  handles and  $b$  boundaries in the presence of the constant background  $B$  field and a compact direction perpendicular to it. When there is the constant background  $B$  field, the open string metric  $G^{\mu\nu}$  and the noncommutativity parameter  $\theta^{\mu\nu}$  are related to the corresponding closed string quantities via

$$G^{\mu\nu} = (g_{\mu\nu} + B_{\mu\nu})_S^{-1} \quad , \quad \theta^{\mu\nu} = 2\pi\alpha'(g_{\mu\nu} + B_{\mu\nu})_A^{-1} \quad , \quad (2)$$

where the subscripts  $S$  and  $A$  denote the symmetric and the antisymmetric parts of a matrix,

respectively. In the absence of the compact direction, the computation of the partition functions was performed in [14, 15] and we will use their results in this note. The new element here will be the incorporation of the topological sectors resulting from the existence of the compact direction.

For the description of worldsheets, it is convenient to first consider the  $(g0)$  worldsheets. On a  $(g0)$  worldsheet, there are  $2g$  homology cycles forming a basis,  $a_\alpha$  and  $b_\alpha$  ( $\alpha = 1, \dots, g$ ) with canonical intersection pairings, and  $2g$  Abelian differentials  $\omega_\alpha$  (holomorphic) and  $\bar{\omega}_\alpha$  (antiholomorphic). These Abelian differentials are normalized along the  $a_\alpha$ -cycles and, when integrated over  $b_\alpha$  cycles, determine the  $g \times g$  period matrix  $\tau$

$$\oint_{a_\alpha} \omega_\beta = \delta_{\alpha\beta} \quad , \quad \oint_{b_\alpha} \omega_\beta = \tau_{\alpha\beta} . \quad (3)$$

Up to three loops, it is known that the moduli space of the worldsheets are parameterized by the symmetric period matrix without any redundancy. In this closed string setup, it is well known how to incorporate the effects caused by a compact target space direction [16]. We find it useful to review the conventional derivations of such effects. The Euclideanized string partition function in the path integral formalism contains a factor

$$\int \mathcal{D}\psi \mathcal{D}\tau \mathcal{D}h \mathcal{D}X \exp(-S_X) \times \dots , \quad (4)$$

where  $\mathcal{D}\psi$ ,  $\mathcal{D}\tau$ ,  $\mathcal{D}h$  and  $\mathcal{D}X$  represent the ghost, the moduli, the worldsheet metric and the target space coordinate  $X$  integrations. The action  $S_X$  in conformal gauge is given by

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{h} h^{\alpha\beta} \partial_\alpha X \partial_\beta X = \frac{i}{2\pi\alpha'} \int dz \wedge d\bar{z} \partial X \bar{\partial} X . \quad (5)$$

Here we have introduced the local holomorphic and antiholomorphic coordinates  $z$  and  $\bar{z}$ . We assume that  $X$  is compactified to a circle with a radius  $R$  via the identification  $X \simeq X + 2\pi R$ . The topological sector of  $X$  field can be written as

$$dX = p^\alpha \omega_\alpha + \bar{p}^\alpha \bar{\omega}_\alpha + \text{fluctuation modes} , \quad (6)$$

where  $p^\alpha$  and  $\bar{p}^\alpha$  are independent holomorphic and antiholomorphic zero mode parameters for closed strings (modulo the overall level matching conditions). These parameters are determined by the holonomy properties under the shift along each cycle:

$$\begin{aligned} \oint_{a_\alpha} dX &= p_\alpha + \bar{p}_\alpha = 2\pi R n_\alpha \\ \oint_{b_\alpha} dX &= \tau_{\alpha\beta} p^\beta + \bar{\tau}_{\alpha\beta} \bar{p}^\beta = 2\pi R m_\alpha , \end{aligned} \quad (7)$$

where  $n_\alpha$  and  $m_\alpha$  are integers. Once these are determined, the topological sector contribution to the partition function can be computed by evaluating the action Eq. (5)

$$\mathcal{Z}_t^{(g)c} = \sqrt{\kappa} \sum_{n_\alpha, m_\alpha} \exp \left[ -\kappa \left( m_\alpha (\text{Im } \tau)^{-1}{}^{\alpha\beta} m_\beta - 2m_\alpha ((\text{Im } \tau)^{-1} \text{Re } \tau)^{\alpha\beta} n_\beta \right. \right. \quad (8)$$

$$\left. \left. + n_\alpha (\bar{\tau} (\text{Im } \tau)^{-1} \tau)^{\alpha\beta} n_\beta \right) \right],$$

by using the formula that holds for a worldsheet without boundaries

$$\int dz \wedge d\bar{z} \partial X \bar{\partial} X = \sum_\alpha \left( \oint_{a_\alpha} dz \partial X \oint_{b_\beta} d\bar{z} \bar{\partial} X - \oint_{a_\alpha} d\bar{z} \bar{\partial} X \oint_{b_\beta} dz \partial X \right), \quad (9)$$

and  $\kappa = \pi R^2 / \alpha'$ . The multiplicative factor  $\sqrt{\kappa}$  in Eq. (8) comes from the center-of-mass part functional integration. Hereafter and in Eq. (8) we ignore the moduli- and  $\kappa$ -independent multiplicative factor in front of the partition function. Using the Poisson resummation formula

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{m=-\infty}^{\infty} \int_{-\infty}^{\infty} dx f(x) e^{2\pi i m x}, \quad (10)$$

we can rewrite Eq. (8) as follows

$$\mathcal{Z}_t^{(g)c} = \kappa^{(1-g)/2} \sqrt{\det (\text{Im } \tau)} \quad (11)$$

$$\times \sum_{m_\alpha, n_\alpha} \exp \left[ -\kappa n_\alpha (\text{Im } \tau)^{\alpha\beta} n_\beta - \frac{\pi^2}{\kappa} m_\alpha (\text{Im } \tau)^{\alpha\beta} m_\beta + 2\pi i m_\alpha (\text{Re } \tau)^{\alpha\beta} n_\beta \right],$$

which makes the T-duality invariance  $R \rightarrow \alpha' / R$  ( $\kappa \rightarrow \pi^2 / \kappa$ ),  $n_\alpha \rightarrow m_\alpha$  and  $m_\alpha \rightarrow n_\alpha$  manifest in the exponential part. In particular, for the torus with  $g = 1$  and  $b = 0$ , one gets

$$\mathcal{Z}_t^{(1)c} = \sqrt{\tau_2} \sum_{m,n=-\infty}^{\infty} \exp \left[ -\kappa \tau_2 n^2 - \frac{\pi^2}{\kappa} \tau_2 m^2 + 2\pi i \tau_1 m n \right], \quad (12)$$

where the torus moduli  $\tau = \tau_1 + i\tau_2$ .

We now turn to the case of our main interest, the  $g = 1$  and  $b = 1$  worldsheets, which correspond to the nonplanar two-loop vacuum worldsheets with the Euler characteristic  $\chi = -1$ . A (11) worldsheet can be considered as the ‘folded’ version of a (20) worldsheet by an anticonformal involution  $I$ , the fixed points of which becoming the boundary. As such, there are two intersecting homology cycles  $a = a_1 - b_2$  and  $b = b_1$  in the homology basis where the period matrix is given by<sup>1</sup>

$$\tau_{\alpha\beta} = \begin{pmatrix} iT_{11} & \frac{1}{2} + iT_{12} \\ \frac{1}{2} + iT_{12} & iT_{22} \end{pmatrix}. \quad (13)$$

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<sup>1</sup>For the conventions of homology cycles, period matrix, etc., we follow those of [15]. One notes that, while the overbar of [15] represents the action of the involution  $I$ , the overbar in this note denotes the complex conjugation.

This period matrix has three independent components  $T_{11}$ ,  $T_{12}$  and  $T_{22}$  corresponding to three moduli parameters of the (11) surfaces. Of the original six moduli parameters of the (20) surfaces, the “even” sector satisfying the condition  $\tau = I(\tau)$  survives the folding operation  $I$ . When compared to another two-loop worldsheet (03) with  $\chi = -1$ , a planar vacuum worldsheet, that does not have intersecting homology cycles, the key difference is the existence of the real part in the off-diagonal elements of the period matrix. The analog of Eq. (6) for (11) surfaces is

$$dX = p^\alpha \omega_\alpha + p^\alpha \bar{\omega}_\alpha + \text{fluctuation modes} , \quad (14)$$

where the index  $\alpha$  runs over  $(1, 2)$ . Since the compact direction in our consideration is perpendicular to the directions where the  $B$  fields are turned on, we impose the usual Neumann boundary condition for  $X$ . This boundary condition, in turn, sets the condition  $p_\alpha = \bar{p}_\alpha$ . While the number of independent  $p$ ’s reduces from four to two, the number of independent cycles also reduces from four to two under the folding operation from (20) surfaces. The analog of the holonomy properties Eqs. (7) becomes

$$\begin{aligned} \oint_a dX &= \oint_{a_1-b_2} (p^\alpha \omega_\alpha + p^\alpha \bar{\omega}_\alpha) = p_1 = 2\pi \tilde{R}n \\ \oint_b dX &= \oint_{b_1} (p^\alpha \omega_\alpha + p^\alpha \bar{\omega}_\alpha) = p_2 = 2\pi \tilde{R}m , \end{aligned} \quad (15)$$

where we specified the holonomy properties under the shifts along the  $a$  and  $b$  cycles and used the explicit form of the period matrix, Eq. (13), for the evaluation of the integrals. Here  $n$  and  $m$  are integers, and  $\tilde{R}$  is the radius of the compact direction. The notable difference between the nonplanar two-loop (11) surfaces and the planar two-loop (03) surfaces is that, for the latter with the non-intersecting homology cycles  $a = b_1$  and  $b = b_2$ , the holonomy properties similar to Eqs. (15) do not constrain the values of  $p_\alpha$ , since the period matrix is purely imaginary unlike the case of the former, Eq. (13). Inserting Eq. (14) to the open string action of the form Eq. (5), we compute the topological sector contribution to the partition function

$$\mathcal{Z}_t^{(2)} = \sqrt{\tilde{\kappa}} \sum_{n,m} \exp \left[ -\tilde{\kappa} (T_{11}n^2 + 2T_{12}nm + T_{22}m^2) \right] , \quad (16)$$

where  $\tilde{\kappa} = 2\pi \tilde{R}^2 / \alpha'$ . When deriving Eq. (16), we used an identity similar to Eq. (9), which is valid for (11) worldsheets,

$$\int dz \wedge d\bar{z} \partial X \bar{\partial} X = \oint_a dz \partial X \oint_b d\bar{z} \bar{\partial} X - \oint_a d\bar{z} \bar{\partial} X \oint_b dz \partial X , \quad (17)$$

for the  $X$  satisfying the equations of motion and the Neumann boundary condition under which the possible boundary term contribution in Eq. (17) vanishes. Upon using the Poisson resummation formula Eq. (10) for the  $n$ -summation, we obtain

$$\mathcal{Z}_t^{(2)} = \frac{1}{\sqrt{T_{11}}} \sum_{n,m} \exp \left[ -\tilde{\kappa} \frac{T_{11}T_{22} - T_{12}^2}{T_{11}} m^2 - \frac{\pi^2}{\tilde{\kappa}} \frac{1}{T_{11}} n^2 - 2\pi i \frac{T_{12}}{T_{11}} mn \right]. \quad (18)$$

The (11) partition function along the uncompactified directions  $X^1, \dots, X^p$  (including the massive mode contributions from the compactified direction  $X^0$ ) when we turn on the  $B$ -field was computed in [14, 15] for noncommutative  $Dp$ -branes. We note that the ghost sector does not change under the influence of the compactification and the background  $B$  field [14]. The (11) partition function can be understood as resulting from the ‘connected’ nonplanar two-point open string insertions along each boundary of a one-loop (02) annulus. For simplicity, we take  $\theta_{12} = -\theta_{21} = \theta$  while setting the open string metric  $G_{11} = G_{22} = 1$ . We then have the partition function [15]

$$\begin{aligned} \mathcal{Z}_\theta^{(2)} = & \sum_I \int dp_1 \int dp_2 \cdots \int dT_{11} \int dT_{22} \int dT_{12} a_I \frac{|W_1(iT_{11})|}{T_{11}^{p/2}} \\ & \times \exp \left[ \cdots - 2\pi\alpha' T_{22}(p_1 p_1 + p_2 p_2 + p_3 p_3 \cdots + M_I^2) \right. \\ & \left. + 2\pi\alpha' \frac{T_{12}^2}{T_{11}}(p_1 p_1 + p_2 p_2 + p_3 p_3 + \cdots) - \frac{1}{2\pi\alpha'} \frac{\theta^2}{4T_{11}}(p_1 p_1 + p_2 p_2) \right], \end{aligned} \quad (19)$$

where  $W_1$  is constructed from the one-loop eta function and the summation over  $I$  goes over the intermediate string mass states running around the connected (external) vertex insertions. The last term in the exponential function of Eq. (19) is the contribution from the stretched strings [11], which is responsible for the UV/IR mixing. In Eq. (19), the two-loop moduli parameter  $T_{12}$  can be interpreted as the separation distance between two vertices along the imaginary axis of the worldsheet. The moduli  $T_{22}$  corresponds to the ‘length’ of the connected leg between two vertex insertions. From [14, 15], one notes that Eq. (19) can be rewritten as

$$\mathcal{Z}_\theta^{(2)} = \int dT_{11} dT_{22} dT_{12} \frac{|W(\tau)|}{\sqrt{\det(2\pi\alpha' G_{\mu\nu} \text{Im } \tau + \frac{i}{2} \theta_{\mu\nu} \mathcal{I})}}, \quad (20)$$

where the  $\text{Im } \tau$  and intersection matrix  $\mathcal{I}$  are defined as

$$(\text{Im } \tau)_{\alpha\beta} = \begin{pmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{pmatrix}, \quad \mathcal{I}_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \det(\text{Im } \tau) = T_{11}T_{22} - T_{12}^2, \quad (21)$$

and  $W(\tau)$  is given by

$$|W(\tau)| = \prod_{a=1}^{10} |\theta_a(0|\tau)|^{-2} \quad (22)$$

and  $\theta_a$ 's are the ten even Riemann theta functions for the (20) surfaces. The indices  $\alpha$  and  $\beta$  run over  $(1, 2)$  and the index  $\mu$  and  $\nu$  run over the noncompact directions  $1, \dots, p$ . The full string partition function is then given by the integral in Eq. (20) with the factor Eq. (18) inserted in the integrand:  $\mathcal{Z}^{(2)} = \mathcal{Z}_\theta^{(2)} \mathcal{Z}_t^{(2)}$ .

Written in the form of Eqs. (18) and (19), it is straightforward to derive the nonplanar two-loop contribution to the partition function of the noncommutative  $\phi^4$  theory in four dimensions. We consider bosonic D3-branes setting  $p = 3$  in Eq. (19) and take the decoupling limit, where we keep open string quantities such as  $\theta$  and  $G_{\mu\nu}$  fixed, while taking the string length scale to zero. We also keep

$$2\pi\alpha'T_{\alpha\beta} = t_{\alpha\beta} \quad (23)$$

fixed, while we take the limit  $\alpha' \rightarrow 0$ . Eq. (23) is an essential scaling in the known string theory computations to recover the noncommutative field theory results [11]. In particular, this scaling exponentially suppresses the massive string mode contributions in Eq. (19) via, schematically,  $\exp(-N_I T) = \exp(-(N_I/2\pi\alpha')t)$  for the excitation number  $N_I$  state, except the leading tachyon mass that we analytically continue to a finite positive  $M^2$  [17]. For the zero mode parts in Eq. (19), we observe that the powers of  $\alpha'$  are just right to allow the replacement of  $T$ 's in the exponential function with  $t$ 's. The interesting part is the topological sector in Eq. (18). For the topological sector to give non-trivial *finite* contributions in the decoupling limit, we have to require that  $\tilde{\kappa}/\alpha'$  be kept fixed in the  $\alpha' \rightarrow 0$  limit, which implies that  $\tilde{R} \rightarrow 0$  in the same limit. This consideration leads us to define the ‘dual’ radius  $\beta$  via

$$\tilde{R} = \frac{\alpha'}{\beta} \rightarrow \tilde{\kappa} = \frac{2\pi\alpha'}{\beta^2}, \quad (24)$$

where we keep  $\beta$  fixed as we take the  $\alpha' \rightarrow 0$  limit. The simple scaling in Eq. (24) has significant implications. We note that the form of Eq. (16), which is equivalent to Eq. (18), is precisely the summation over the quantized loop momenta  $m/\beta$  and  $n/\beta$

$$\sum_{m,n} \exp \left[ -2\pi\alpha' \left( \frac{n/\beta}{m/\beta} \right)^T \text{Im } \tau \left( \frac{n/\beta}{m/\beta} \right) \right] \quad (25)$$

along the two intersecting cycles present in (11) worldsheets [14, 15] moving in the compact target space direction. To summarize, since the decoupling limit involves the  $\tilde{R} \rightarrow 0$  limit, the string theory temperature should be inversed to be the noncommutative field theory temperature:  $T_{\text{field}} = (1/T_{\text{string}}) \times (1/\alpha')$ . In this process, the ‘winding’ mode description Eq. (15) naturally transmutes to the ‘momentum’ mode description Eq. (25). The scaling Eq. (24) should be



universally taken for an arbitrary values of  $p$  and low energy couplings. Specific to the  $\phi^4$  theory, we further set  $T_{12} = 0$  in the zero mode parts so that the quartic interaction vertices are produced in the low energy Feynman diagram description. In addition, we choose the mass  $M$  of the resulting particle to be very small by taking the limit  $\beta M \ll 1$ , concentrating on the high temperature regime. Using the two integrals

$$\int_0^\infty dt_{11} \frac{1}{t_{11}^2} \exp\left(-\frac{A}{t_{11}}\right) = \frac{1}{A} \quad , \quad \int_0^\infty dt_{22} \exp(-Bt_{22}) = \frac{1}{B} \quad , \quad (26)$$

we immediately find that

$$\mathcal{Z}^{(2)} = -\frac{F^{(2)}}{T} = g^2 \sum_{n,m} \int dp_1 dp_2 dp_3 \frac{1}{\left(\frac{m^2}{\beta^2} + (p_1^2 + p_2^2 + p_3^2)\right) \left(\pi^2 n^2 \beta^2 + \frac{1}{4} \theta^2 (p_1^2 + p_2^2)\right)} \quad , \quad (27)$$

where  $g^2$  is the coupling constant of the  $\phi^4$  theory. The expression Eq. (27) is precisely the noncommutative field theory result obtained in [5].

As spelled out earlier, the field theory limit partition function shows stringy natures when  $\theta \neq 0$ . We note that the  $X^1$  and  $X^2$  part of the partition function Eq. (20) contains the weight factor

$$\frac{1}{t_{11} t_{22} - t_{12}^2 + \theta^2/4} \quad . \quad (28)$$

Therefore, the moduli integration over  $t_{11} t_{22} - t_{12}^2$  variable has the main contribution in the regime where  $t_{11} t_{22} - t_{12}^2 \simeq \theta^2$ . Plugging this into Eq. (18), we find that

$$\mathcal{Z}_t^{(2)} \simeq \sum_{n,m} \frac{1}{\sqrt{t_{11}}} \exp \left[ -\frac{\theta^2}{\beta^2} \frac{1}{t_{11}} m^2 - \pi^2 \beta^2 \frac{1}{t_{11}} n^2 - 2\pi i \frac{t_{12}}{t_{11}} m n \right] \quad , \quad (29)$$

becoming similar to the torus partition function Eq. (12) if we identify  $\tau_2 = 1/t_{11}$  and  $\tau_1 = -t_{12}/t_{11}$ . This fact is not surprising when one considers the open string UV factorization channel where the boundary of (11) surfaces shrinks to small size. It is consistent with the general observation that noncommutative field theories retain the string theory topological sector information [14, 15]. Furthermore, Eq. (29) is invariant under a ‘duality transformation’

$$\beta \rightarrow \frac{1}{\pi} \frac{\theta}{\beta} \quad , \quad n \rightarrow m \quad , \quad m \rightarrow n \quad , \quad (30)$$

formally similar to the T-duality of closed string theory, except that the string scale  $\alpha'$  is replaced by the noncommutativity parameter  $\theta$ . This behavior is reminiscent of the idea advocated in [8] that, in the noncommutative field theory, the noncommutativity scale  $\theta$  plays the role of the effective string scale  $\alpha'$  in the UV limit. We note that while the arguments of [8] are based on

the dual supergravity analysis (a strong coupling argument), our analysis is perturbative. The closed string T-duality is realized both perturbatively and non-perturbatively. Physically, the  $\theta$  appearing in Eq. (28) plays the role of a UV (small  $t$ ) cutoff and it also corresponds to the effective stretching size of ‘loop stretched strings’. Near the stretching size, or the UV cutoff length, in the moduli space, it appears that there exists a kind of ‘duality’ where the noncommutativity scale plays the role of the effective string scale. In fact, one’s natural expectation is that the scale  $\theta$  is the minimum length scale that appears in the commutator  $[X^1, X^2] = \theta$  [1, 8].

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